Chapter 3

3-3.

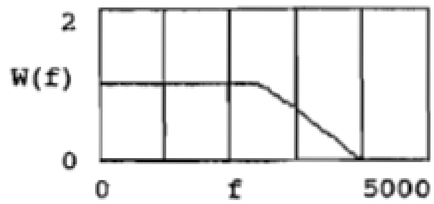
A := 1 f1 := 2500 f2 := 4000

f := 0,200 ..5000

$$W1(x) := \text{if}(|x| < f1, A, 0)$$

$$W2(x) := \left[\frac{-A}{f2 - f1}\right]\cdot(|x| - f2)\cdot(\Phi(|x| - f1) - \Phi(|x| - f2))$$

$$W(x) := W1(x) + W2(x)$$



$$fs := 10000 \qquad \tau := 50\ 10^{-6} \qquad Ts := \frac{1}{fs} \qquad d := \frac{\tau}{Ts}$$

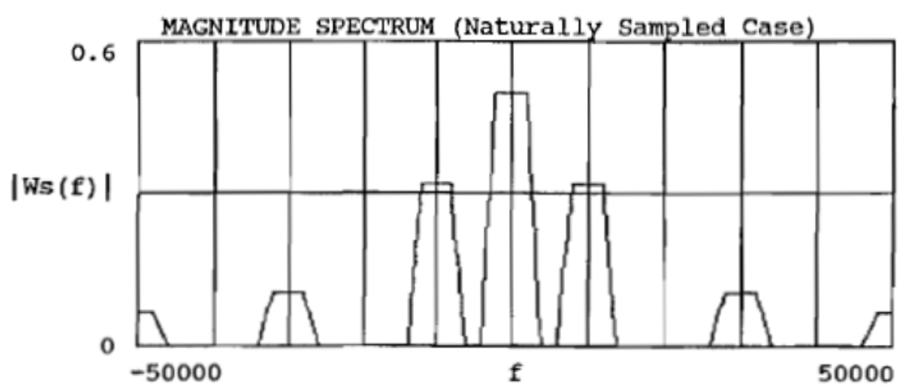
(a) Naturally-sampled PAM

n := -5,-4 ..5

$$Sa(x) := \text{if}\left[x \neq 0, \frac{\sin(x)}{x}, 1\right]$$

f := -50000,-48000 ..50000

$$Ws(f) := d\cdot\sum_{n}(Sa(\pi\ n\ d))\cdot W(f - n\cdot fs)$$



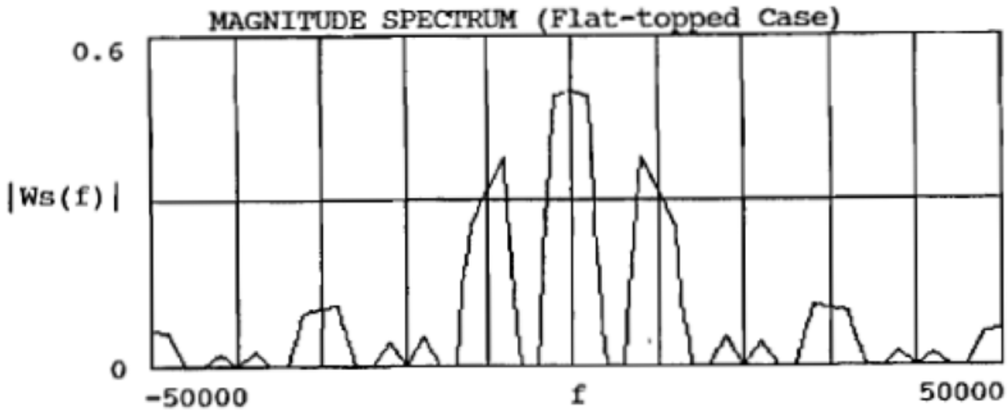
3-3 Cont'd.

(b) Flat-topped PAM

$$H(f) := \tau \, Sa(\pi \cdot \tau \, f)$$

$$Ws(f) := \left[\frac{1}{Ts}\right] \cdot H(f) \cdot \sum_{n} W(f - n\, fs)$$

See next screen for plot

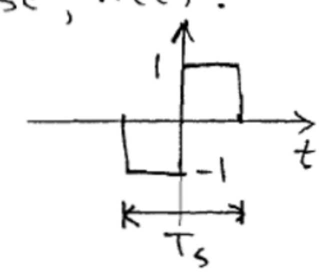


$\boxed{3-6.}$ Using (3-10)

$$W_s(f) = \frac{H(f)}{T_s} \sum_{k=-\infty}^{\infty} W(f - kf_s)$$

where $H(f)$ is the spectrum of the Manchester encoded pulse, $h(t)$.

Thus



$$H(f) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt$$

$$= \int_{-T_s/2}^{0} (-1) e^{-j\omega t} dt + \int_{0}^{T_s/2} (1) e^{-j\omega t} dt$$

3-6 Cont'd.

$$= \frac{j}{\omega}\left[-e^{-j\omega t}\Big|_{-T_s/2}^{0} + e^{-j\omega t}\Big|_{0}^{T_s/2} \right]$$

$$= \frac{-j}{\omega}\left[2 - 2\left(\underbrace{\frac{e^{j\omega T_s/2} + e^{-j\omega T_s/2}}{2}}_{\cos\frac{\omega T_s}{2}} \right)\right]$$

$$H(f) = -j T_s \frac{\left(1 - \cos \omega T_s/2\right)}{\omega T_s/2}$$

```
A := 1        f1 := 2500      f2 := 4000

                                            f := 0,200 ..5000
W1(x) := if(|x| < f1,A,0)

W2(x) := [ -A / (f2 - f1) ] (|x| - f2) (Φ(|x| - f1) - Φ(|x| - f2))

W(x) := W1(x) + W2(x)
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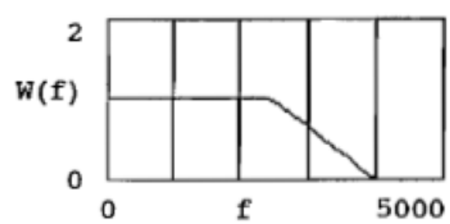


3-6 Cont'd.

$$fs := 10000 \qquad Ts := \frac{1}{fs}$$

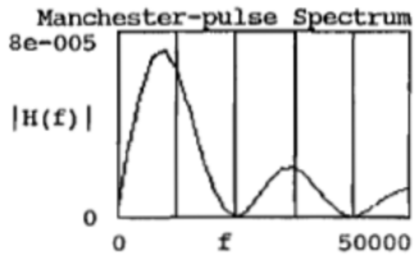
$$j := \sqrt{-1}$$

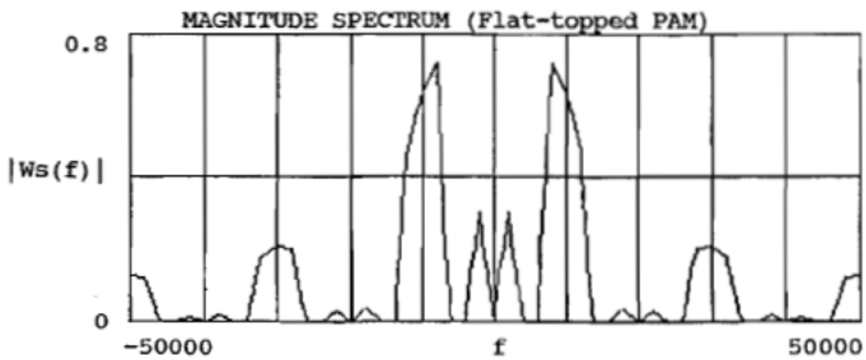
$$n := -5,-4\ ..5 \qquad Sa(x) := \text{if}\left[x \ne 0, \frac{\sin(x)}{x}, 1\right]$$

$$f := -50000,-48000\ ..50000$$

$$H(f) := j \cdot Ts \cdot Sa\left[\pi \cdot f \frac{Ts}{2}\right] \cdot \sin\left[2\ \pi \cdot f \cdot \frac{Ts}{4}\right]$$

$$Ws(f) := \left[\frac{1}{Ts}\right] \cdot H(f) \cdot \sum_n W(f - n\ fs)$$





3-8. (a.) $f_s = 2B = 2(100) = \underline{\underline{200 \text{ samples/sec}}}$

(b.) Using the results given in prob. 3-7.

$$n \ge 3.32 \log_{10}\left(\frac{50}{P}\right) = 3.32 \log_{10}\left(\frac{50}{0.1}\right)$$

$$= 8.96$$

$$\underline{\underline{n = 9 \text{ bits/word}}}$$

(c.) $R = \left(\frac{n \text{ bits}}{\text{word}}\right)\left(\frac{f_s \text{ words}}{\text{sec}}\right) = 200(9) = \underline{\underline{1.8 \frac{\text{k bits}}{\text{sec}}}}$

(d.) For binary PCM $D = R$

eq. (3-74) $D = \frac{2B}{1+r}$, for B_{min} , $r = 0$

$$\Rightarrow \quad B = \frac{D}{2} = \underline{\underline{900 \text{ Hz}}}$$

3-11.

(a.) $f_s \geq 2B_{analog} = 2(20\ kHz) = 40\ \frac{k\ samples}{sec}$

For 8X oversampling of the recovered PCM signal
(used to increase f_s 8X and simplify LPF requirements)

$\ni f_{8x} = 8 f_s = 320\ k\ samples/sec$

$B_{null} = R = n f_{8x} = \left(16 \frac{bits}{sample}\right)\left(320 \frac{k\ samples}{sec}\right) = \underline{\underline{5.12\ MHz}}$

(b) Using (3-18)

$\left(\frac{S}{N}\right)_{peak} = 6.02n + 4.77dB = 6.02(15) + 4.77 = \underline{\underline{94.77dB}}$

3-14. (a.) $P_e = 10^{-4}$ $\quad \frac{S}{N} \geq 30\ dB$

Eq. (3-16) $\left(\frac{S}{N}\right)_{dB} = 10 \log_{10}\left[\frac{3M^2}{1+4(M^2-1)P_e}\right]$

$M = 2^n$ levels

for $n=4$: $\left(\frac{S}{N}\right)_{OUT} = 28.4\ dB$

for $\underline{\underline{n=5}}$: $\left(\frac{S}{N}\right)_{OUT} = 33.4\ dB$

$\longrightarrow M = 2^5 = \underline{\underline{32}}$ levels

(b.) $f_s = 2(2.7\ KHz) = 5.4\ k\ \frac{samples}{sec}$

The first zero-crossing of the $\frac{\sin x}{x}$ type spectrum is :

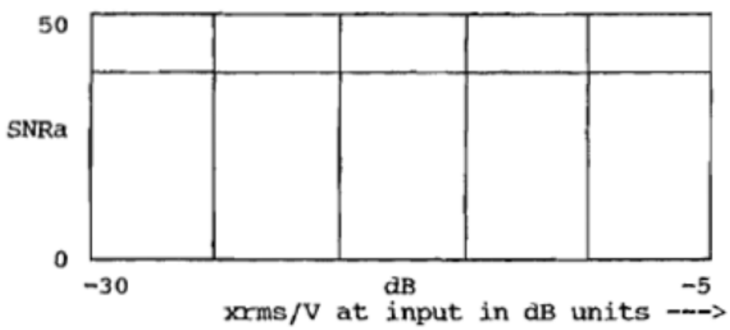
$B = \frac{n}{T_s} = n f_s = 5(5.4k) = \underline{\underline{27\ KHz = B}}$

3-18.

dB := -30,-29 ..-5 M := 256 $n := \frac{\log(M)}{\log(2)}$

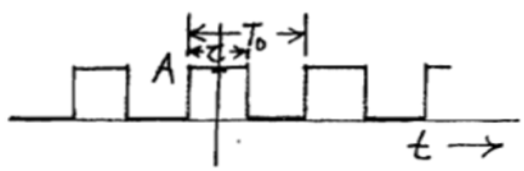
μ := 255 n = 8

SNRa := 6.02 n + 4.77 − 20 log(ln(1 + μ))



3-21.

For alternating data the waveform is periodic where $T_0 = 2T_b$.



From (2-109) the spectrum is

$$W(f) = \sum_{-\infty}^{\infty} c_n\, \delta(f - nf_0)$$

Where $c_n = \frac{1}{T_0}\int_a^{a+T_0} w(t)\, e^{-jn\omega_0 t}\,dt = \frac{1}{T_0}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} A\, e^{-jn\omega_0 t}\,dt$

$$= \frac{A}{T_0}\, \frac{e^{-jn\omega_0 t}}{-jn\omega_0}\Bigg|_{-\frac{\tau}{2}}^{\frac{\tau}{2}} = \frac{2A}{T_0}\underbrace{\left(\frac{e^{-jn\omega_0\tau/2} + e^{+jn\omega_0\tau/2}}{-2jn\omega_0}\right)}_{\sin(n\omega_0\tau/2)}$$

$$\Rightarrow c_n = \frac{\cancel{2}A\tau}{T_0\cancel{2}}\,\frac{\sin(\omega_0 n\tau/2)}{\frac{n\omega_0\tau}{2}} = \frac{A}{2}\left(\frac{\tau}{T_b}\right)\frac{\sin\left(\frac{n\pi}{2}\frac{\tau}{T_b}\right)}{\left(\frac{n\pi}{2}\frac{\tau}{T_b}\right)}$$

$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2T_b}$

$$\Rightarrow W(f) = \sum_{n=-\infty}^{\infty} \frac{A}{2}\left(\frac{\tau}{T_b}\right)\left(\frac{\sin\left(\frac{n\pi}{2}\frac{\tau}{T_b}\right)}{\left(\frac{n\pi}{2}\frac{\tau}{T_b}\right)}\right)\delta\left(f - \frac{n}{2}R\right) \qquad (A)$$

where $R = \frac{1}{T_b}$ = bit rate

3-21. Cont'd

(a) Using (A) for NRZ signaling with $\tau = T_b$

$$|W(f)| = \sum_{-\infty}^{\infty} \frac{A}{2} \left| \frac{\sin\left(\frac{n\pi}{2}\right)}{\left(\frac{n\pi}{2}\right)} \right| \delta\left(f - \frac{n}{2}R\right)$$

Unipolar NRZ (alternating data)

If the data are a sequence of four "1"'s followed by four "0"'s, the waveform would have the same shape except T_0 would be 4 times as large. i.e. $T_0 = 8T_b$.

$$\Rightarrow |W(f)| = \sum_{-\infty}^{\infty} \frac{A}{2} \left| \frac{\sin\left(\frac{n\pi}{2}\right)}{\left(\frac{n\pi}{2}\right)} \right| \delta\left(f - \frac{n}{8}R\right)$$

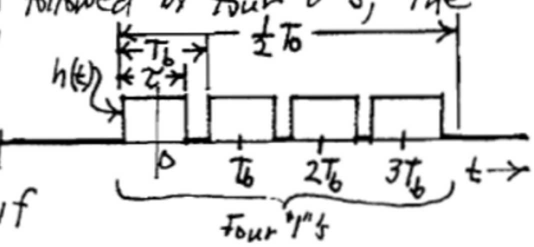
Unipolar NRZ 4 "1"'s plus 4 "0"'s

(b) Using (A) for RZ signaling with $\tau = \frac{3}{4}T_b$,

$$|W(f)| = \sum_{-\infty}^{\infty} \frac{3}{8}A \left| \frac{\sin\left(\frac{3}{8}n\pi\right)}{\left(\frac{3}{8}n\pi\right)} \right| \delta\left(f - \frac{n}{2}R\right)$$

Unipolar RZ (alternating data)

For RZ with four "1"'s followed by four "0"'s, the periodic waveform would appear as shown where $T_0 = 8T_b$. The mathematical calculations are simplified if (2-112) is used



$$c_n = f_0\, H(nf_0)$$

where $h(t)$ is the basic waveform that is repeated to create the periodic waveform (as shown in the figure). $h(t)$ consists of the superposition of four rectangular pulses. Using the time delay theorem of Table 2-1

3-21 (b). Cont'd.

and the rectangular pulse spectrum of Table 2-2

$$H(f) = A\tau \frac{\sin(\pi f \tau)}{\pi f \tau}\left[1 + e^{-j\omega T_b} + e^{-j\omega 2T_b} + e^{-j\omega 3T_b}\right]$$

or

$$c_n = \frac{A\tau}{8T_b}\,\frac{\sin\left(\frac{n\pi}{8}\frac{\tau}{T_b}\right)}{\left(\frac{n\pi}{8}\frac{\tau}{T_b}\right)}\left[1 + e^{-j\frac{n\pi}{4}} + e^{-j\frac{n\pi}{2}} + e^{-j\frac{3}{4}n\pi}\right]$$

$$\boxed{f = nf_0 = \frac{n}{T_0} = \frac{n}{8T_b}}$$

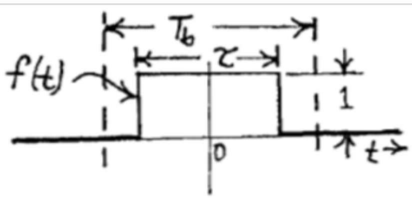
For RZ with $\tau = \frac{3}{4}T_b$, this becomes

$$c_n = \frac{3}{32}A\left(\frac{\sin\left(\frac{3}{32}n\pi\right)}{\left(\frac{3}{32}n\pi\right)}\right)\left[1 + e^{-j\frac{n\pi}{4}} + e^{-j\frac{n\pi}{2}} + e^{-j\frac{3}{4}n\pi}\right]$$

Thus, the spectrum for Unipolar RZ with four alternate "1" and "0"'s is

$$|W(f)| = \sum_{n=-\infty}^{\infty} \frac{3}{32}A\left|\frac{\sin\left(\frac{3}{32}n\pi\right)}{\left(\frac{3}{32}n\pi\right)}\right|\left|1 + e^{-j\frac{n\pi}{4}} + e^{-j\frac{n\pi}{2}} + e^{-j\frac{3}{4}n\pi}\right|\delta\left(f - \frac{n}{8}R\right)$$

$\boxed{\text{3-24.}}$ (a) Substituting (3-40) into (3-36a) the PSD for Polar RZ signaling is



$$\mathcal{P}(f) = \frac{A^2}{T_b}\,|F(f)|^2$$

where the pulse shape, $f(t)$, is shown in the figure. Thus,

$$F(f) = \mathcal{F}[f(t)] = \tau\,\frac{\sin(\pi f \tau)}{(\pi f \tau)}$$

and

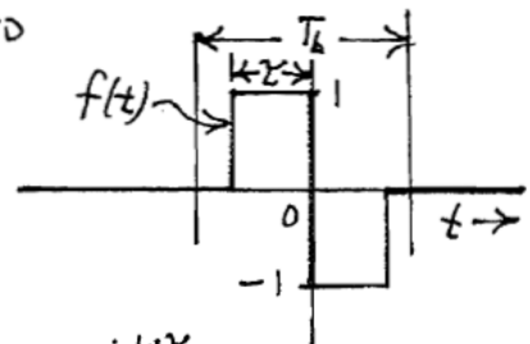
$$\mathcal{P}(f) = \frac{A^2\tau^2}{T_b}\left[\frac{\sin(\pi f \tau)}{(\pi f \tau)}\right]^2$$

For the case of $\tau = \frac{1}{2}T_b$, this becomes

$$\underline{\underline{\mathcal{P}(f) = \frac{A^2 T_b}{4}\left[\frac{\sin(\frac{\pi}{2} f T_b)}{(\frac{\pi}{2} f T_b)}\right]^2}}$$

The first-null bandwidth is $B_{null} = \frac{2}{T_b} = 2R$
and the bandwidth efficiency is $\eta = \frac{1}{2}$ (bit/sec)/Hz.

(b.) Equation (3-36) can also be used to evaluate the PSD for RZ Manchester signaling where the pulse shape is shown in the figure.



$$F(f) = \tau\left(\frac{\sin(\pi f \tau)}{(\pi f \tau)}\right)\left[e^{j\omega\frac{\tau}{2}} - e^{-j\omega\frac{\tau}{2}}\right]$$

$$\Rightarrow \quad F(f) = j2\,\tau\left(\frac{\sin(\pi f \tau)}{(\pi f \tau)}\right)\sin\left(\frac{\omega\tau}{2}\right)$$

Using (3-40) and (3-36), the PSD for Manchester signaling is

$$\mathcal{P}(f) = \frac{4A^2\tau^2}{T_b}\left[\frac{\sin(\pi f \tau)}{(\pi f \tau)}\right]^2 [\sin(\pi f \tau)]^2$$

If $\tau = \frac{1}{4}T_b$, this becomes

$$\underline{\underline{\mathcal{P}(f) = \frac{1}{4}A^2 T_b\left[\frac{\sin(\frac{\pi}{4} f T_b)}{(\frac{\pi}{4} f T_b)}\right]^2 \left[\sin(\tfrac{\pi}{4} f T_b)\right]^2}}$$

The first-null bandwidth is $B_{null} = \frac{4}{T_b} = 4R$
and the spectral efficiency is $\eta = \frac{1}{4}$ (bits/sec)/Hz.

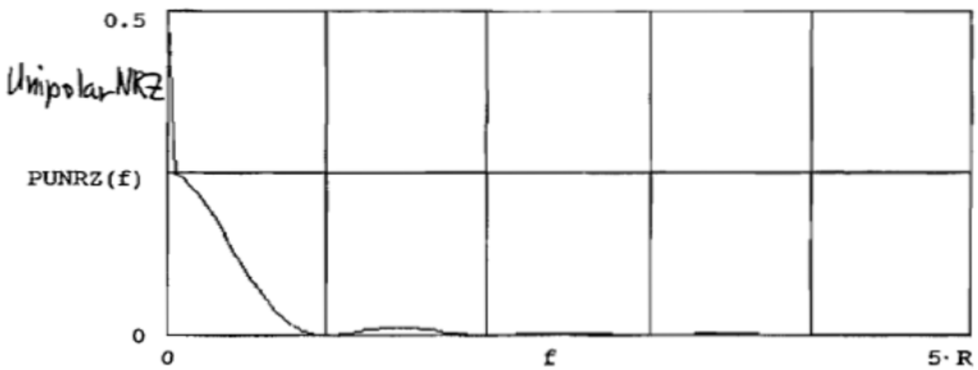
3-26. A := 1 R := 1 f := 0,0.05 ..5 $Tb := \frac{1}{R}$

$$Sa(x) := if\left[x \neq 0, \frac{\sin(x)}{x}, 1\right]$$

The PSD for Unipolar NRZ is given by (3-39b) and consists of both a continuous spectrum and a discrete spectrum. The computer cannot plot *infinite* values for the delta functions, so plot the weights of the delta functions instead. Thus (3-39b) will be broken into two functions, one for the continuous spectral plot and one for the discrete spectral plot.

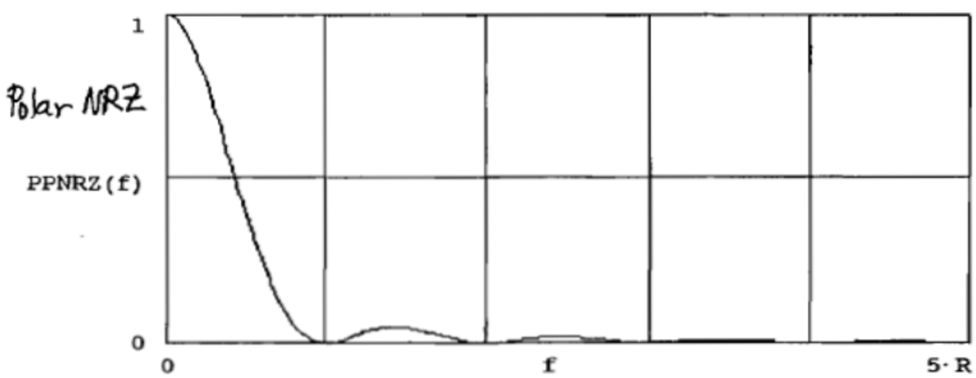
$$PUNRZc(f) := \left[A^2 \cdot \frac{Tb}{4}\right] \cdot (Sa(\pi\ f\ Tb))^2 \qquad PUNRZd(f) := if\left[f \neq 0, 0, \frac{A^2}{4}\right]$$

$$PUNRZ(f) := PUNRZc(f) + PUNRZd(f)$$



Use (3-41) for Polar NRZ spectrum:

$$PPNRZ(f) := \left[A^2\ Tb\right]\ (Sa(\pi\ f \cdot Tb))^2$$



The PSD for Unipolar RZ is given by (3-43) and consists of both a continuous spectrum and a discrete spectrum. The computer cannot plot infinite values for the delta functions, so plot the weights of the delta functions instead. Thus (3-43) will be broken into two functions, one for the continuous spectral plot and one for the discrete spectral plot.

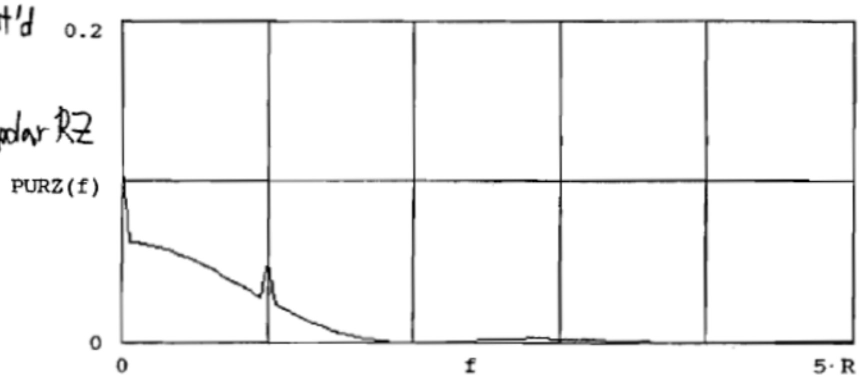
$$PURZc(f) := \left[A^2\ \frac{Tb}{16}\right]\ \left[Sa\left[\pi\ f\ \frac{Tb}{2}\right]\right]^2$$

$$PURZd(f) := if\left[mod(f,R) \neq 0, 0, \frac{A^2}{16} \cdot \left[Sa\left[\pi\ f\ \frac{Tb}{2}\right]\right]^2\right]$$

3-26 Cont'd

PURZ(f) := PURZc(f) + PURZd(f)

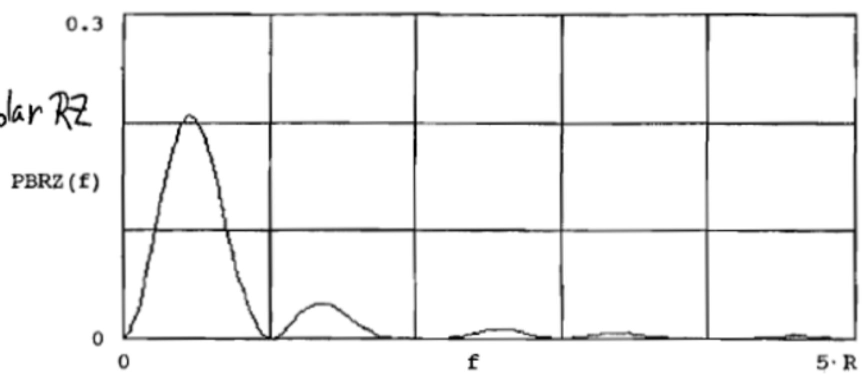
Unipolar RZ



Using (3-45) the PSD for Bipolar RZ is:

$$PBRZ(f) := A^2 \left[\frac{Tb}{4}\right] \cdot \left[Sa\left[\pi\ f \cdot \frac{Tb}{2}\right]\right]^2 \ (\sin(\pi\ f\ Tb))^2$$

Bipolar RZ



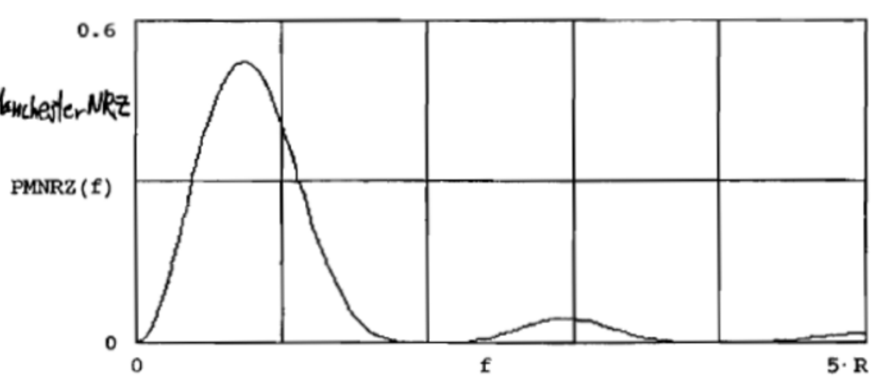
$A := 1 \qquad R := 1 \qquad f := 0, 0.05 \,..5 \qquad Tb := \frac{1}{R}$

$$Sa(x) := if\left[x \neq 0, \frac{\sin(x)}{x}, 1\right]$$

Use (3-46c) for the Manchester NRZ spectrum:

$$PMNRZ(f) := A^2 \cdot Tb \left[Sa\left[\pi \cdot f \cdot \frac{Tb}{2}\right]^2\right] \cdot \left[\sin\left[\pi\ f \cdot \frac{Tb}{2}\right]^2\right]$$

Manchester NRZ



3-34. Use the result from Prob 3-7.

(a) $n \geq 3.32 \log_{10}\left(\frac{50}{P}\right) = 3.32 \log_{10}(50) = 5.64 \Rightarrow$ Use $n = 6$ bits/word.

$f_s = 2B = 5.4\text{ kHz} \Rightarrow R_{min} = nf_s = 6(5.4\text{ kHz}) = \underline{\underline{32.4\text{ kbits/sec}}}$

(b) $L = B = 2^\ell \Rightarrow \ell = 3$ bits D/A $\quad D = \frac{R}{\ell} = \frac{32.4\text{ kbit/sec}}{3\text{ bits/symbol}} = \underline{\underline{10.8\text{ ksym/sec}}}$

(c) $D = \frac{2B}{1+r}$ where $r = 0$ for min BW $\Rightarrow B = \frac{D}{2} = \underline{\underline{5.4\text{ kHz}}}$

3-35. $L = B = 2^\ell \Rightarrow \ell = 3$

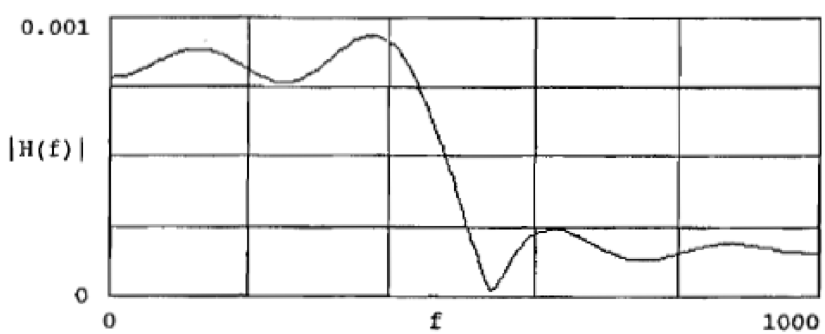
(a) $D = \frac{R}{\ell} = \frac{9600\text{ bits/sec}}{3\text{ bits/symbol}} = \underline{\underline{3.2\text{ k symbol/sec}}}$

(b) $D = \frac{2R}{1+r} = \frac{2(2.4k)}{1+r} = 3.2k \Rightarrow \underline{\underline{r = 0.5}}$

3-38. (a.)

f := 0,10 ..1000 $\qquad$ fs := 1000

$$H(f) := \int_0^{0.008} \frac{\sin(\pi \cdot fs \cdot (t - 0.004))}{\pi\ fs\ (t - 0.004)} e^{-2j \cdot \pi\ f\ t}\, dt$$



(b.) From the figure above, the bandwidth for the causal approximation is $\underline{\underline{B = 540\text{ Hz}}}$

The bandwidth for the noncausal filter is $B = \frac{1}{2} f_s = \underline{\underline{500\text{ Hz}}}$

3-42. $h_e(t) = \int_{-\infty}^{\infty} H_e(f) e^{j2\pi ft} df$

For $t = nT_s \Rightarrow h_e(nT_s) = \int_{-\infty}^{\infty} H_e(f) e^{j2\pi nT_s f} df$

Break into multiple integrals, each with a $\frac{1}{T_s}$ wide interval.

$$\Rightarrow h_e(nT_s) = \sum_{k=-\infty}^{\infty} \int_{-\frac{1}{2T_s}+\frac{k}{T_s}}^{\frac{1}{2T_s}+\frac{k}{T_s}} H_e(f) e^{j2\pi nT_s f} df$$

Let $f_1 = f - \frac{k}{T_s}$

Then $$h_e(nT_s) = \sum_{k=-\infty}^{\infty} \int_{-\frac{1}{2T_s}}^{\frac{1}{2T_s}} H_e\left(f_1 + \frac{k}{T_s}\right) e^{j2\pi nT_s\left(f_1 + \frac{k}{T_s}\right)} df_1$$

or
$$h_e(nT_s) = \int_{-\frac{1}{2T_s}}^{\frac{1}{2T_s}} \sum_{k=-\infty}^{\infty} H_e\left(f_1 + \frac{k}{T_s}\right) e^{j2\pi nT_s f_1} df_1$$

Assume $\sum_{-\infty}^{\infty} H_e\left(f_1 + \frac{k}{T_s}\right) = T_s, \quad |f_1| < \frac{1}{2T_s}$

Then
$$h(nT_s) = \int_{-\frac{1}{2T_s}}^{\frac{1}{2T_s}} T_s e^{j2\pi nT_s f_1} df_1 = \left. \frac{\cancel{T_s}\, e^{j2\pi nT_s f_1}}{j2\pi n\cancel{T_s}} \right|_{-\frac{1}{2T_s}}^{\frac{1}{2T_s}}$$

or
$$h(nT_s) = \frac{e^{j\cancel{2}\pi n\cancel{T_s}\frac{1}{\cancel{2T_s}}} - e^{-j\cancel{2}\pi n\cancel{T_s}\frac{1}{\cancel{2T_s}}}}{j2\pi n} = \frac{\sin(n\pi)}{n\pi} = \begin{Bmatrix} 1, & n=0 \\ 0, & n\neq 0 \end{Bmatrix}$$

Q.E.D.

$\boxed{3\text{-}45.}$ $M = 16 = 2^4 \Rightarrow n = 4$

(a) Binary PCM $\Rightarrow \ell = 1$, $R = nf_s = 4f_s = D$

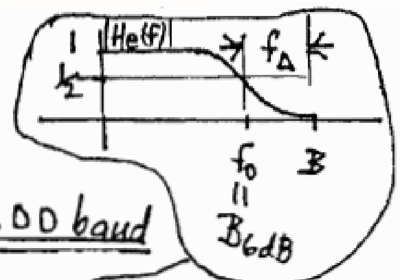
$$D = \frac{2B}{1+r} = \frac{2(4\,kHz)}{1+0.5} = \underline{\underline{5.33\ k\,bits/sec}}$$

(b) From (a) $f_s = \frac{D}{4} = \frac{5.33k}{4} = 1.33\,kHz$

$$B_{\substack{analog\\max}} = \frac{f_s}{2} = \frac{1.33k}{2} = \underline{\underline{667\ Hz}}$$

$\boxed{3\text{-}47.}$ (a) $L = 2^\ell = 4 \Rightarrow \ell = 2$

$$D = R/\ell = \frac{2400}{2} = \underline{\underline{1200\ baud}}$$



(b) $B = \frac{1}{2}(1+r)D$ where $r = \frac{f_\Delta}{f_0} = 0 \Rightarrow B = f_0 = B_{6dB}$

$$\Rightarrow B_{6dB} = \frac{1}{2}(1+0)D = \frac{1}{2}(1200) = \underline{\underline{600\ Hz}}$$

(c)

$$B_{absolute} = \frac{1}{2}(1+r)D = \frac{1}{2}(1+0.5)(1200) = \frac{3}{4}(1200)$$

$r = 0.5$

$$\Rightarrow \underline{\underline{B_{absolute} = 900\ Hz}}$$

3-53.

(a.) From (3-84)

$$\delta = \frac{2\pi f_a A}{f_s} \; ; \quad f_a = 3.4\,\text{kHz} \;\; \& \;\; A = \frac{1}{2}$$

We need to determine the f_s which the Channel can support. Assuming that a $r=0$ roll-off factor is used, then

$$f_s = D = 2B = 2(1\,\text{MHz}) = 2 \times 10^6 \,\frac{\text{Samples}}{\text{sec}}$$

$$\Rightarrow \; \delta = \frac{2\pi(3.4k)\left(\frac{1}{2}\right)}{2 \times 10^6} = \underline{\underline{0.00534}}$$

(Note: The channel has to be equalized with a Nyquist filter.)

(b.)

$$\delta = \frac{2\pi(3.4k)\left(\frac{1}{2}\right)}{25 \times 10^3} = \underline{\underline{0.427}}$$

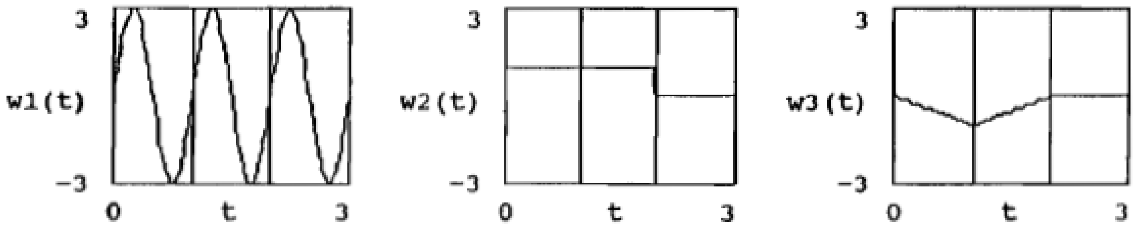
(Note: No Channel equalization required.)

3-55.

$T := 0.05 \qquad t := 0, 0.05 .. 3$

$w1(t) := 3 \cdot \sin(2 \cdot \pi \cdot t) \qquad w2(t) := \mathrm{if}((|t| - 1) \le 1, 1, 0)$

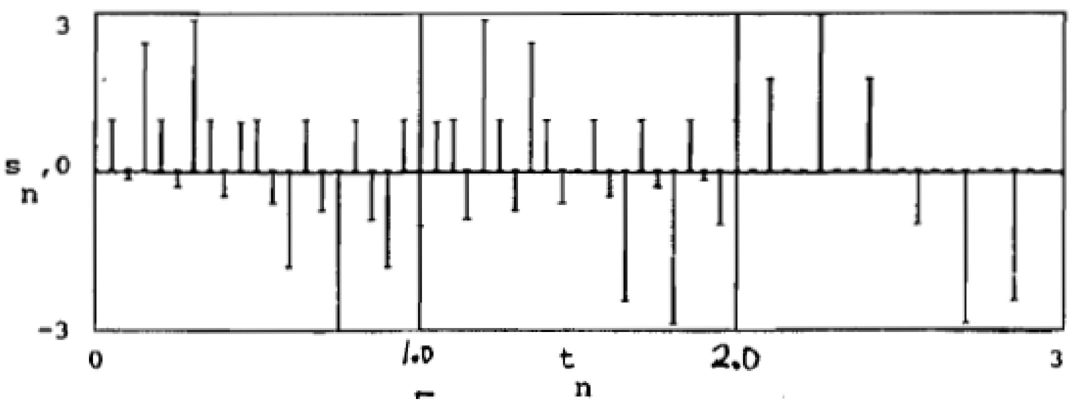
$w3(t) := \mathrm{if}(|t - 1| \le 1, |t - 1| - 1, 0)$



$n := 0 .. 62 \qquad t_n := n \cdot T \qquad k := 0 .. 20$

$k1(k) := 3 \cdot k \qquad k2(k) := 3 \cdot k + 1 \qquad k3(k) := 3 \cdot k + 2$

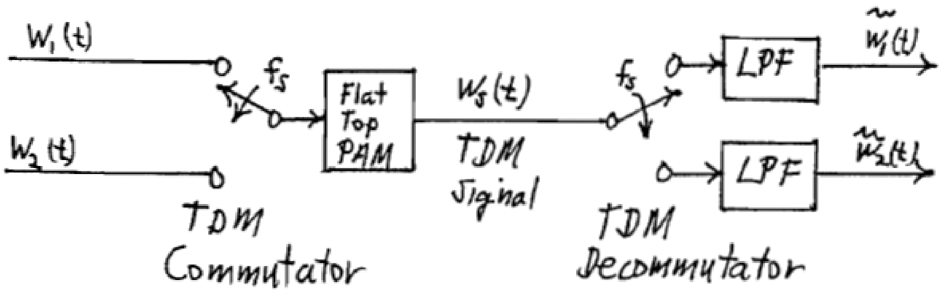
$s_{k1(k)} := w1\left[t_{k1(k)}\right] \qquad s_{k2(k)} := w2\left[t_{k2(k)}\right] \qquad s_{k3(k)} := w3\left[t_{k3(k)}\right]$



3-57. (a.) Each analog signal has a highest frequency of $B = 3\,kHz$

$\Rightarrow$ The minimum sampling frequency for each analog signal is $\underline{\underline{f_s = 2B = 6\,kHz}}$

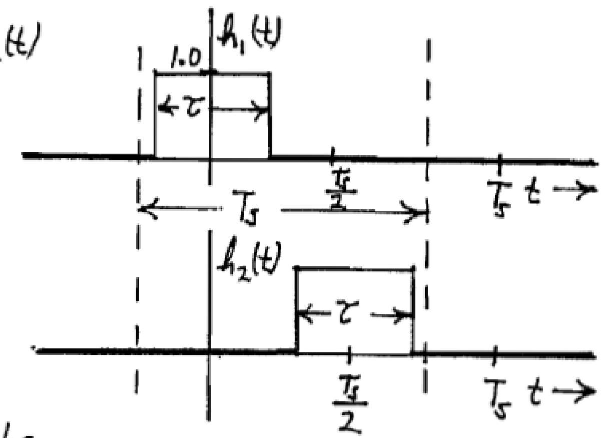
3-57 .(a.) Cont'd



(b.) Referring to (3-8), the sampled TDM signal is

$$w_s(t) = \sum_{k=-\infty}^{\infty} w_1(kT_s)\,h_1(t-kT_s) + \sum_{k=-\infty}^{\infty} w_2(kT_s)\,h_2(t-kT_s)$$

where $h_1(t)$ and $h_2(t)$ are shown in the figure and $\tau \le \frac{T_s}{2}$ and $f_s \ge 2B$.



Following the same procedure as described in (3-8) thru (3-13), the spectrum of the TDM instantaneously sampled (flat-topped) PAM signal is

$$W_s(f) = \frac{1}{T_s} H_1(f) \sum_{k=-\infty}^{\infty} W_1(f-kf_s) + \frac{1}{T_s} H_2(f) \sum_{k=-\infty}^{\infty} W_2(f-kf_s)$$

where $H_1(f) = \tau\left(\frac{\sin(\pi f \tau)}{\pi f \tau}\right)$ and $H_2(f) = \tau\left(\frac{\sin(\pi f \tau)}{\pi f \tau}\right) e^{-j2\pi f \frac{T_s}{2}}$

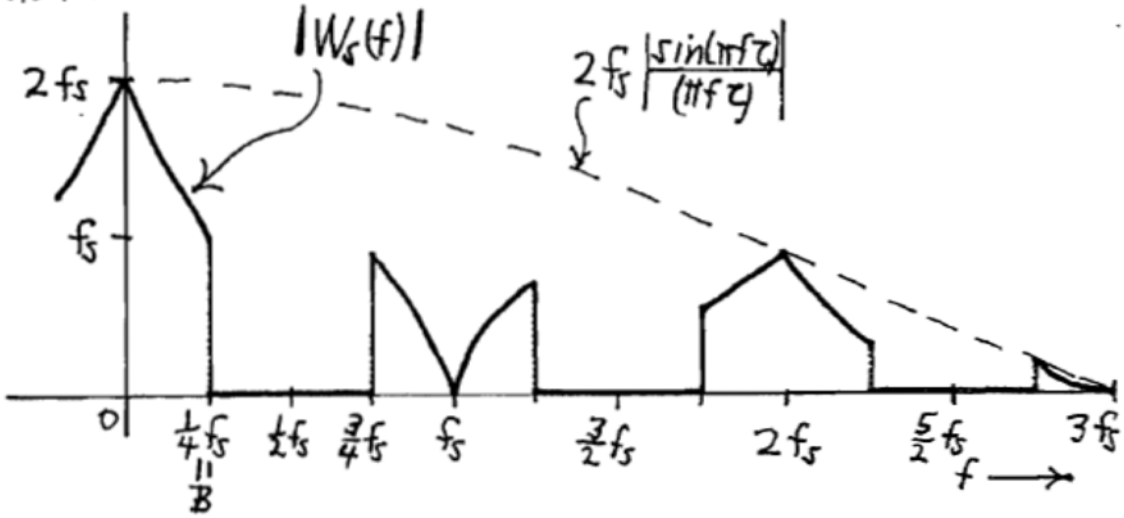
$$\Rightarrow W_s(f) = f_s\,\tau\,\frac{\sin(\pi f \tau)}{\pi f \tau} \sum_{k=-\infty}^{\infty} \Pi\left(\frac{f-kf_s}{2B}\right) + 2B\tau\left(\frac{\sin(\pi f \tau)}{(\pi f \tau)}\right)$$

$$+ 2B\tau\left(\frac{\sin(\pi f \tau)}{(\pi f \tau)}\right) e^{-j\pi T_s f} \sum_{k=-\infty}^{\infty} \Lambda\left(\frac{f-kf_s}{B}\right)$$

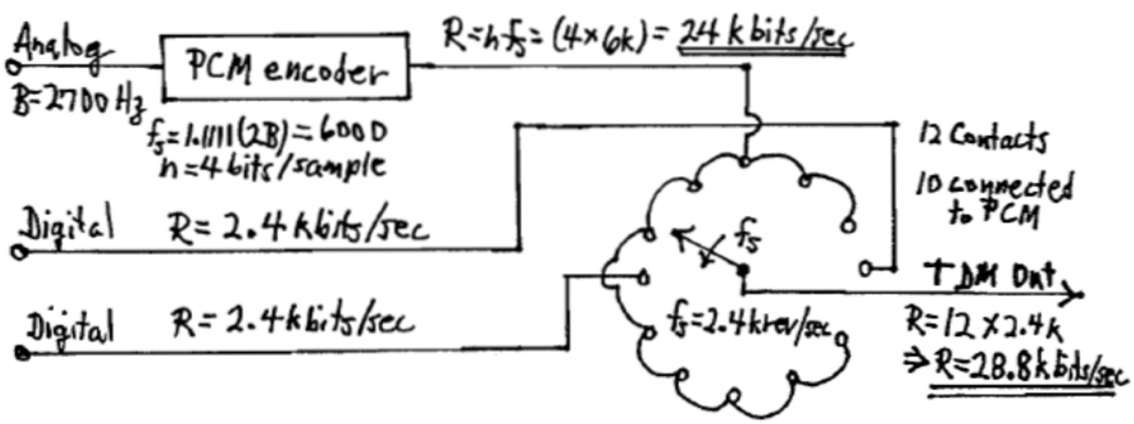
3-57. (b.) Cont'd

Thus,
$$|W_s(f)| = f_s \left|\frac{\sin(\pi f \tau)}{(\pi f \tau)}\right| \sum_{k=-\infty}^{\infty} \left| \Pi\left(\frac{f-kf_s}{2B}\right) + e^{j\pi T_s f} \Lambda\left(\frac{f-kf_s}{B}\right)\right|$$

For the sketch, let the parameters be the same as those shown in Fig. 3-6. Let $\tau/T_s = 1/3$, $f_s = 4B$. Using a programable calculator, the following sketch is obtained.



3-60.



3-64.

(a) For PCM a $N=8$ dimensional system is used since any of the 256 messages can be represented by

$$s_i(t) = \sum_{j=1}^{8} s_{ij}\, \varphi_j(t)$$

where $s_{ij} = \pm 1$ for binary PCM.

$$T_0 = \frac{1}{10 \text{ mess/sec.}} = \underline{\underline{0.1 \text{ sec/message}}}$$

$$B = \frac{1}{2}\left(\frac{N}{T_0}\right) = \frac{1}{2}\left(\frac{8}{0.1}\right) = \underline{\underline{40 \text{ Hz}}}$$

(b) For PPM a $N=256$ dimensional system is used:

$$s_i(t) = \sum_{j=1}^{256} s_{ij}\, \varphi_j(t) \quad \text{where } s_{ij} = \delta_{ij}$$

$$B = \frac{1}{2}\left(\frac{N}{T_0}\right) = \frac{1}{2}\left(\frac{256}{0.1}\right) = \underline{\underline{1,280 \text{ Hz}}}$$